

|  | $\checkmark$ | Reason abstractly and quantitatively. (MP.2) | $\checkmark$ | Attend to precision. (MP.6) |
| :---: | :---: | :---: | :---: | :---: |
|  | $\checkmark$ | Construct viable arguments and critique the reasoning of others. (MP.3) | $\square$ | Look for and make use of structure. (MP.7) |
|  | $\checkmark$ | Model with mathematics. (MP.4) | $\square$ | Look for and express regularity in repeated reasoning. (MP.8) |
|  | LEARNER OUTCOME(S) <br> - Students will compare payment options expressed in a variety of units and representational forms. <br> - Students will apply the ECPC (Estimate-Convert-PlotCompare) strategy for solving complex price comparison tasks. |  | ASSESSMENT TOOLS/METHODS <br> - Steps 5 and 6 will serve as evidence of student mastery. <br> o During Step 5, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. If students are working alone, the teacher should have students think out loud as they solve the problem (similar to how the teacher modeled the ECPC strategy out loud in Step 3). <br> o During Step 6, allow students the opportunity to modify their solutions based on what they learn from watching others present their solutions. <br> - Exit Slip: You are trying to decide between three brands of cereal. Reechios is on sale for $\$ 2.79$ a box. Flosted Frakes cost $\$ 3.69$ a box, but have a "buy 2, get 1 free sale." Organic Carrot Flakes are $\$ 5.89$ a box, but are $50 \%$ off. All the boxes are the same weight. Which cereal is the best deal if you want to buy 2 or 3 boxes? Explain your reasoning. |  |
|  |  | - Students should know how to estimate prices (in dollars), convert between units, construct 2-variable line graphs, and calculate percentages. |  | ert between units, construct 2-variable line graphs, and |



## RESOURCES

Student copies of ECPC Strategy handout (attached)
Student copies of Discount Evaluation Task handout (attached)

Student copies of Water Evaluation Task handout (attached)

Student copies of Car Comparison Task handout (attached)

Teacher Answer Sheet (attached)
Vocabulary Sheet (attached)



|  | TECHNOLOGY INTEGRATION <br> Students can compare discounts for places they actually shop at the following site: http://www.retailmenot.com These two articles <br> provide information on comparing lease vs. purchase of a car (much more in-depth than the lesson went). <br> http://www.ehow.com/about 7330025 car-lease-vs -buying-cheaper .html <br> http://www.insure.com/car-insurance/vehicle-lease-information.html <br> PURPOSEFUL/TRANSPARENT <br> Spending money wisely is an important concern for students. This lesson asks students to use the ECPC strategy in three situations <br> (comparing discounts, comparing modes of transportation, comparing purified water sources). |
| :--- | :--- |
| CONTEXTUAL <br> Real life price comparison contexts are used throughout the lesson. Although they are simplified for the purpose of the course, <br> many of the complex elements inherent in price comparisons still remain. Moreover, the comparative discount website provided is <br> an actual site that is excellent for saving money, and most of the prices used for the tasks are actual prices. |  |
| BUILDING EXPERTISE <br> Students synthesize their knowledge of simple percentage calculations and unit conversions in complex multi-step tasks. The final <br> step of plotting the graphs forges connections between algebraic and graphical representations. |  |

## Rolling in the Money: Vocabulary Sheet

2-variable line graphs - a line graph that plots the dependent variable as a function of the independent variable on the $\mathrm{X}-\mathrm{Y}$ plane.

High cognitive level - mathematical processing where students make connections between different representations (e.g., graphs and algebra), or where they engage in true understanding of a problem rather than just following a routine.

Thinkaloud technique - a teaching practice where the instructor (or student) attempts to verbalize all of his or her thoughts in solving a problem.

Unit conversions - translating a value from one measurement unit into another (such as meters to feet or meters to centimeters).

## ECPC Strategy

stimate. Both in real life situations and on standardized tests, we often do not have time to make every calculation precisely, especially in a multi-step process with messy numbers. At its most basic level, estimation is about balancing accuracy with ease. If the students feel they can multiply, divide, add, and subtract the numbers quickly, there is no need to sacrifice accuracy for ease. For example, $\$ 3+\$ 9+\$ 11=?$. We could round each number to the nearest $\$ 5$ and get $\$ 25$, or the nearest $\$ 10$ and get $\$ 20$, but it is just as easy to add the actual numbers and get the exact answer of $\$ 23$. On the other hand, $\$ 15.95$ * 4 = ? would be much easier as $\$ 16$ * 4 = $\$ 64$ and even simpler (but a bit less accurate) as $\$ 15$ * 4 = $\$ 60$. How do you know when you've estimated too much? In a test situation, if your answer comes very close to two of the possible choices, you should probably go back and check your estimation. When you are comparing four or more things to find the best option (like when you go shopping), you can use a rough estimate to determine the best two options and then a good estimate or exact calculation to decide which is the better of the two.
onvert. There are many types of conversions. There are the standard measurement conversions (length, volume, speed) and then there are conversions of discounts. For example, how could you convert "half-off" into something that could be compared with a $30 \%$ discount? You could convert the $30 \%$ to three-tenths off, or you could convert half-off into $50 \%$. In fact, if you feel more comfortable with decimals, you could convert them both to decimals ( $50 \%=.50$ and $30 \%=.30$ ). Although the Convert step comes after the Estimate step, you can always use estimation to make your conversion easier. For example, if you are trying to convert 16 kilograms to pounds and you know that $1 \mathrm{~kg}=2.2 \mathrm{lbs}$, you might estimate that 16 kilograms equals about 32 pounds $(16 * 2=32)$.

lot. In a simple estimation or conversion, you will probably not need to plot. However, for complicated problems, a nice graph can make your life a lot easier. It can also show you relationships that you never knew existed. Consider a situation at the grocery store where there is a "Buy 3 , get 1 free" promotion on Woochies grape juice. Perhaps buying 1 or 2 jugs of the generic brand would be cheaper than 1 or 2 jugs of Woochies, but with the discount, buying 4 jugs of Woochies is better than 4 jugs of the generic brand. In this case, your comparison would depend on how many items you are buying. This can get very complicated, but plotting a graph lets you make comparisons at a glance.

ompare. Once you plot the graph, the hard work is done. In real life, the numbers are just one factor of a comparison. Buying generic grape juice might be cheaper, but perhaps you crave that distinct aftertaste that Woochies leaves in your mouth. The plot allows you to compare different options using numbers, but it is up to you to then factor in everything else you consider important. Of course, in a test situation, you will only be asked to compare the numerical portion.

## Discount Evaluation Task

Ever since the economic crisis, you have been looking for any possible way to save money. You decided to start buying your basic home products in bulk online, and then discovered that you can save even more money by using internet coupon codes. At the moment, you are ready to buy some value packs of toilet paper, and are trying to decide which of the following five codes to use (you can only choose one). In all cases, except where the regular price is noted, assume that the price for 1 value pack of toilet paper is $\$ 9.44$, and that shipping is a base rate of $\$ 2.99$ plus $\$ .99$ for each pack. You would prefer to buy somewhere between 2 and 5 packs, depending on how you can get the best deal. All prices include tax.

1. Construct a chart or graph that would be effective in helping you make a decision.
2. Determine which coupon code you think represents the best deal and explain why.

## ACTIVE COUPONS



## Car Comparison Task

You're looking for some transportation to get to work, and it appears that you have numerous options. Assume that all prices already include taxes, and that parking is free at your place of residence.

1. Construct a chart or graph that would be effective in helping you make a decision.
2. Determine which transportation system would be most cost-effective over the course of 2-5 years

Option 1. Lease a new car

You can lease a 2012 Coyote Tamarack for \$329 per month with no down payment. You may return at any yearly mark (i.e. after 1 year, 2 years, 3 years, etc.). Because you do not actually own the car, Coyote requires the you purchase collision, liability, and gap insurance for a total of $\$ 88.99$ a month. You estimate that you will spend about $\$ 30 /$ week on gas. You will not be responsible for any repairs.


Option 3. Buy a used car

You've found a reputable dealer offering to sell a 1994 Misty Beast for $\$ 8,000$. He's willing to take off a quarter of the price if you can pay in cash (which you can, since you've been saving and you just received a handsome tax refund). Since you will own the car, you only need to purchase liability insurance, which is $\$ 32 /$ month. You estimate that you will spend about $\$ 30 /$ week on gas. However, you will need to pay more in repairs: about \$300/year.


Option 2. Buy a new car

After researching several options, you find a new car for $\$ 26,899$. Loan payments are $\$ 389$ per month with a down payment of $\$ 1,000$. You estimate that you can sell it at any time, but that the value will continuously decrease at a rate of about 20\% per year. Because you are receiving a loan, you will need to purchase collision and liability insurance for a total of $\$ 80 /$ month. You estimate that you will spend about $\$ 30 /$ week on gas. You do not expect any repairs in the first five years.


Option 4. Public transport (bus)

The bus pass prices are $\$ 62.00$ per month. Assume a yearly inflation rate of $5 \%$ (i.e. at the beginning of each year, the prices are raised by 5\%). If you decide to use a bus instead of a car, you will need to purchase enough passes each month so that your family has transportation also. Since you will want to travel in-between cities occasionally, and to places where buses don't run, you should budget in an extra \$700/year for such trips.


## Water Evaluation Task

You never thought it would come to this, but you are ready to start buying water. After checking the EPA reports for your local area, you decide to go with some form of purified water. Assume that you are making a decision for the next 3 to 6 years, and that your household will be drinking about 2 gallons of water per day. (Remember that 128 ounces $=1$ gallon).

1. Construct a chart or graph that would be effective in helping you make a decision.
2. Determine which clean water source represents the best deal and explain why.

Option 1. Bottled water

Cases of thirty 16.9 ounce bottles of water are sold for \$15 each, plus $6 \%$ sales tax. However, if you buy 3, you get 1 free.


Option 3. Water purification system

A water system built directly into your sink will cost $\$ 1329.37$, regular price, tax included. Filters and maintenance total (with tax) about \$28/year. Right
now, you can receive a 15\%
discount on a system.

## Option 2. Water machine

Agua Limpia will come to your house and install a machine for free, but you must sign a contract to purchase a minimum of four 20-gallon jugs per month. Each jug is \$6.95, tax included. You can cancel the contract at any time with no penalty. Agua Limpia will cover all service fees


Option 4. Purifier jug for the fridge

The jug itself is \$13.98 (with no filters), and 3-packs of filters sell for \$19.99 each. Neither of those amounts include the 6\% sales tax. Also, to avoid bacteria growing inside the filter, you will need to change it every 2 months or 50 gallons, whichever comes first.


## Rolling in the Money: Teacher Answer Sheet

Estimated answers are approximate and may not match the answers provided below exactly.

## Task 1

1. Here's an example of a table with the total price and price per roll for each of the five offers. It would also be fine to create another method of displaying the information (such as a line graph).

|  |  | Number of value packs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 4 | 5 |
| ```Estimated Price (in US$)``` | Offer 1 | 24.00 | 34.50 | 35.50 | 46.00 |
|  |  | 12.00 | 11.50 | 8.88 | 9.20 |
|  | Offer 2 | 24.00 | 28.50 | 38.00 | 47.50 |
|  |  | 12.00 | 9.50 | 9.50 | 9.50 |
|  | Offer 3 | 21.00 | 30.00 | 39.00 | 48.00 |
|  |  | 10.50 | 10.00 | 9.75 | 9.60 |
|  | Offer 4 | 21.00 | 30.00 | 39.00 | 48.50 |
|  |  | 10.50 | 10.00 | 9.75 | 9.70 |
|  | Offer 5 | 19.00 | 29.50 | 35.00 | 45.50 |
|  |  | 9.50 | 9.83 | 8.75 | 9.10 |

2. Answers may vary with student, but should be supported with logic. Gray boxes in the above chart show some of the lowest prices per value pack.

## Task 2

1. Here's an example of a table for each of the four options. In this case, a graph might be more helpful, because it allows students to see trends in the data, as well as connections between algebraic and graphical representation (for options 1, 3, and 4, it is easy to see the linear trend; in option 2, there is an exponential trend).

|  |  | Number of years |  |  |  |
| :---: | :--- | ---: | ---: | ---: | ---: |
|  |  | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| Estimated <br> Cost <br> (in US\$) | Option 1 | 13000 | 19500 | 26000 | 32500 |
|  | Option 2 | 13,901 | 19,682 | 24,868 | 29,567 |
|  | Option 3 | 10200 | 12300 | 14400 | 16500 |


2. Answers may vary with student, but should be supported with logic. Gray boxes in the above chart show some of the lowest prices per value pack.

## Task 3

1. Here's an example of a table for each of the four options. In this case, a graph might be more helpful, because it allows students to see trends in the data, as well as connections between algebraic and graphical representation (for options 1,3 , and 4 , it is easy to see the linear trend; in option 2, there is an exponential trend).

|  |  | Number of years |  |  |  |
| :---: | :--- | ---: | ---: | ---: | ---: |
|  |  | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
|  | Option 1 | 6300 | 8400 | 10500 | 12600 |
| Estimated <br> Cost <br> (in US\$) | Option 2 | 1050 | 1400 | 1750 | 2100 |
|  | Option 3 | 1240 | 1270 | 1300 | 1330 |
|  | Option 4 | 315 | 415 | 515 | 615 |


2. In this case, there is a clear answer; Option 4 is the cheapest. However, 2,3 , and 4 are all reasonably similar, and so students may support their decision with other reasons (besides cost).

